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Numerical homogenization of a fractured rock mass: A geometrical approach to determine the mechanical Representative Elementary Volume

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ABSTRACT

Numerical Homogenization (NH) using finite element method is an indirect and rigorous method allowing the determination of large scale mechanical properties (tensor of elasticity, strength) of a fractured rock mass (Pouya and Ghoreychi, 2001; Min and Jing, 2003; Chalhoub, 2006; Pouya and Chalhoub, 2008). However, in order to obtain a representative equivalent homogenized medium, it is necessary to determine the Representative Elementary Volume (REV). Using NH, this parameter is identified by preliminary iterative calculations of some mechanical properties on a set of increasing domains' size. Moreover, heavy mesh generation and cost computing time are required for each fractured domain especially in the case of finite fractures' length. In this paper we examine if it is possible to determine the mechanical REV from analyses carried out on geometrical parameters, which don't require mesh making. As a case study, a 2D fractured rock mass is analyzed where the mechanical properties of randomly distributed fractures are homogenous. It is shown that the mechanical REV can be deduced from the geometrical REV obtained for the fractures spacing.

KEYWORDS: Representative Elementary Volume (REV) Rock Mass, Numerical Homogenization, Cost computing time, Anisotropic elasticity

NOMENCLATURE

FRM	Fractured rock mass	$U_{ii}(k)$ *[m]*	Average displacement in "i" direction on a side "k" of a square
REV	Representative elementary volume	$F_i(k)$ *[MN]*	Sum of the forces parallel to "i" direction on a side "k" of a square
E [MPa]	Young modulus of a rock	ε_{ii}	Average strain in "i" direction
ν	Poisson ratio of a rock	ε_{ij}	Average shear strain

C [MPa]	Cohesion of a rock	σ_{ii} *[MPa]*	Average stress in "i" direction
Φ [°]	Friction angle of a rock	σ_{ij} *[MPa]*	Average shear stress
k_n *[MPa/m]*	Normal stiffness of fractures	E_{ii} *[MPa]*	Young modulus in "i" direction of a FRM
k_t *[MPa/m]*	Shear stiffness of fractures	ν_{ij}	Poisson ratio due to a load in "j" direction of a FRM
c [MPa]	Cohesion of fractures	G_{ij} *[MPa]*	Shear coefficient of a FRM
ϕ [°]	Friction angle of fractures	L_i *[m]*	REV length in "i" direction
e [m]	Spacing of fractures		

INTRODUCTION

In FRM (fractured rock mass) design projects, such as bridge or dam foundations, slope stability at roads and railroad edges, tunnel and underground galleries; one is sometimes in the presence of a dense and fine fracturing of which the effect must be taken into account in a global way. If the random distribution of the fractures in the rock mass is homogenous at macro-scale, then the mechanical properties can be considered as homogeneous at a sufficiently large scale compared to the fracturing size.

The scale at which the properties can be considered as homogeneous is called Representative Elementary Volume (REV). For the same rock mass the REV size can be different for different mechanical, hydraulic, thermal, geometrical, or other properties. The REV size for the equivalent permeability of a FRM was studied in a pioneer work by Long et al. (1982) and then by Wei et al. 1995, Pouya and Courtois (2002) and Min *et al*. (2004). The existence of an REV for two-phase flow in heterogeneous porous rock was discussed by Ataie-Ashtiani *et al.* (2001). The mechanical REV was studied by Pouya and Ghoreychi (2001) who calculated the equivalent Young modulus for different domain sizes and in this way they determined the Elastic REV size. Then they supposed that this REV size represents the scale at which all the mechanical properties (strength, etc.) can be considered as homogeneous.

It is significant to note that in an anisotropic medium and thus obviously in a rock mass, the size of the REV can be different in the various directions. This property can be easily observed in the anisotropic periodic case or in a FRM with only one family of infinite extension. However, in the case of an anisotropic distribution of finite fractures size, the anisotropy of the REV size was not well discussed in the literature.

Determination of the REV size requires a detailed attention. Different methods existing in the literature allow the determination, with more or less rigorous and simple way, the equivalent homogeneous properties of the FRM. But it should be noted that the use of the homogenized properties in various problems is relevant only if the project size (for example the tunnel diameter or the slope height) is sufficiently large compared to the scale to which the FRM can be regarded as homogeneous.

The homogenized properties can be given in various manners:

"Rock Mass Classification Methods" (Bieniawski, 1973; Barton *et al.*, 1974) determine in an empirical and rather qualitative way, the homogenized properties of the FRM according to the rock matrix properties and certain characteristics of the fracturing. Therefore we can, according to these characteristics, consider an equivalent modulus of elasticity or values of cohesion and angle of friction for the FRM. But these methods do not analyze accurately neither the scale to which the homogenized properties are relevant nor the anisotropy of these properties.

The research of the mechanical properties (elastic and strength) of the FRM by analytical methods (Bekaert and Maghous 1996, Buhan and Maghous 1997, Tsurkov and Kachanov 2000) is limited to the case of infinite fractures extension. Moreover, fractures that belong to the same set must be parallel. In these methods, a limited number of sets can be taken into account.

Using Numerical Homogenization it is possible to determine in a rigorous and indirect way, the large scale mechanical properties (elasticity, strength) of a FRM. In these methods, the fracturing is expressed by various statistical laws describing the orientation, the extension, the density and the thickness of the fractures.

Different sets of fractures are generated in a finite volume (cube). Fractures parameters (orientation, size, thickness, etc.) obey predefined statistical laws. Once the mesh generation is done and in order to calculate the stress strain curves for various types of loading, the Finite Element Method (Pouya and Ghoreychi, 2001; Chalhoub, 2006; Pouya and Chalhoub, 2008) or the Distinct Element Method (Min and Jing, 2003) may be used.

The possibility to determine the REV of a FRM is another advantage of the numerical homogenization. This size represents the minimum scale beyond which the behavior of the FRM and the homogenized medium are equivalent, or the scale from which the properties of the FRM can be considered homogeneous. It is necessary to consider different domains of increasing size (square in the two-dimensional studies). The variation of simple elastic characteristics such as the Young modulus or the Poisson ratio is analyzed according to the domains' size. The minimum size beyond which the statistical fluctuations of these characteristics are negligible is then determined. It represents the REV size for the homogenized medium. It is on larger domain sizes of the REV that more advanced and complete calculations are done to estimate, for instance, the cohesion or the angle of friction of the FRM (Pouya and Ghoreychi, 2001; Chalhoub, 2006; Pouya and Chalhoub, 2008).

The determination of the REV size requires preliminary simulations on different domain sizes. Mesh generation must be done for each domain. As the mesh generation procedures of a FRM are heavy and expensive for the reason of fractures presence which leads to a heavy computing time cost, it would be quite interesting to search for a simpler method to estimate the REV size. The question that arises then naturally is: is is there any geometrical characteristic easy to determine, and especially not requiring mesh, that can inform us about the size of the mechanical REV? It is to this question that we are trying to answer by comparing the mechanical REV to the size of geometrical REV obtained from different geometrical characteristics such as fractures average spacing.

PRESENTATION OF THE FRM

We consider as a study case a granitic FRM in the western south of France (de la Vienne). Numerical filters have been developed to avoid mesh and numerical calculation problems. They made insignificant modifications to the fracturing data of the FRM.

Rock mass fracturing model

Fractures are generated in a 3D volume (cube). They are represented by disks in the space (Baecher 1983, Fig. 1). A 2D representation of the rock mass is considered in this study (Fig. 2). It is obtained by cutting the cube by a vertical plane. The x-axis, in this representation, corresponds to the horizontal North-South direction, and the y-axis, to the vertical direction. The fractures size, orientation, density and aperture or thickness (filled fractures) introduced in this model have been deduced from the geological data. The possible propagation of existing fractures that may locally occur is not considered. This question has a second order effect regarding the intensity of existing fractures and also the insufficient data available especially for the extension of fractures.

The fractures locations are randomly generated in the rock mass. There is no privilege direction to define one or more sets of fractures. The extension of the existing fractures varies from 20cm to 30m and their slope (angle with the x axis) varies from 0° to 90° (clock wise). The statistical distribution of the fractures extension is assumed to obey a decreasing exponential law with an average value of 10m. The orientation obeys a normal distribution law with an average value of 60° (sub-vertical fractures) and a 10° variance. The basic fractures density is uniform and equal to $0.4m^{-2}$. In this study, the real value of the density is less then $0.4m^{-2}$ because of the numerical filters mentioned above. The value of the fractures thickness required for the determination of the joint behaviour is assumed to be about a few millimetres on the basis of some core samples observation. A statistical representation of these fractures is given in Figure 2 for a square domain of a 50m side.

Constitutive models of intact rock and fractures

Laboratory test results show that the intact rock has a linear and isotropic behaviour in the elastic stage, with values of E=72000MPa and ν=0.25. The strength properties of the intact rock are given by a Mohr-Coulomb criteria with c=17MPa and ϕ=57°. The fractures behaviour in the elastic domain is assumed to be linear and characterized by $k_n=4\times10^5$MPa/m and $k_t=3\times10^6$MPa/m. Beyond the elastic domain, Mohr-Coulomb criteria is used with c=1.51MPa and ϕ=27°. The present investigation aims to determine the mechanical REV size based on the analysis of elastic homogenized properties: the elastic domain behaviour of the rock mass will be discussed in coming simulations (§ 4.2).

NUMERICAL EXPERIMENTS

In the basic square (Fig. 2) we cut out square domains of variable sizes (dashed lines) on which we calculated various geometrical and mechanical characteristics. All the squares have the same center and they are parallel to the basic (50m×50m) domain.

Geometrical REV

The studied domains were cut out by straight lines that correspond to different lines of survey. Three different directions were given to this line: parallel to the y axis (vertical survey) parallel to the x axis (horizontal survey) and with 45° (inclined survey). On each line of survey we calculated the number of fractures intersections, and thus the linear density of the fractures. The reverse of this density gives us the average spacing of the fractures.

Figure 3 represents the variation of the average spacing according to the domains size for the three directions of survey. It is noticed that, for a higher size than approximately 15m, the fluctuation becomes negligible. The average spacing attains limit values of 0.7m for the vertical direction, 0.8m for the horizontal direction and 0.5m for the inclined direction. According to this result, we can estimate a value of 15m for the geometrical REV based on the research of the average spacing.

It is noticed that for a relative size higher than 42m, average spacing starts to grow slightly. This increase represent an edge effect perfectly explained and does not have to be taken into account. During the model generation, the fracture that has its center apart from the cube (50m×50m×50m) is eliminated. As the fractures have an average radius of 5m, then their density decreases for a domain size exceeding 45m. Consequently, their average spacing increases.

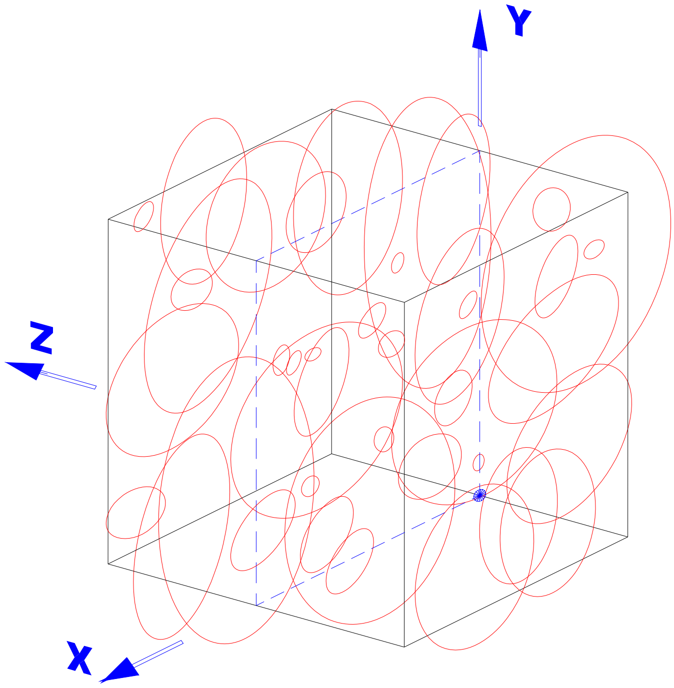


Figure 1: 3D representation of a fractured rock mass

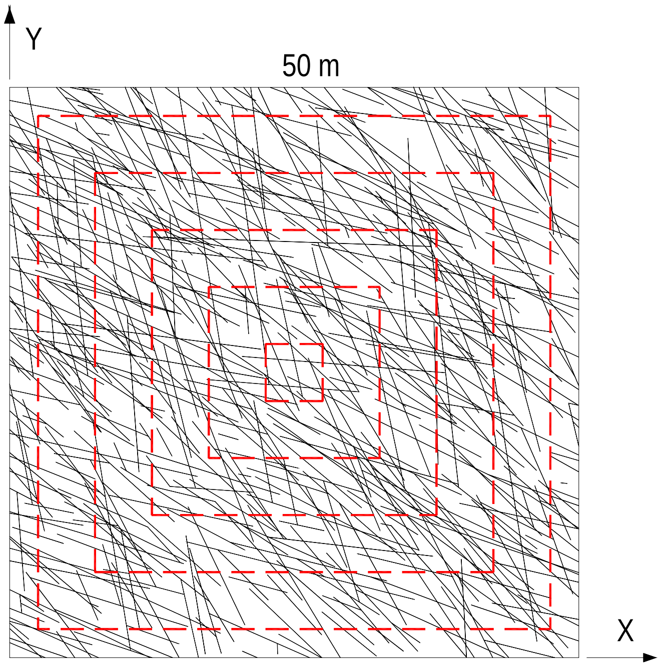


Figure 2: 2D representation of the FRM case study

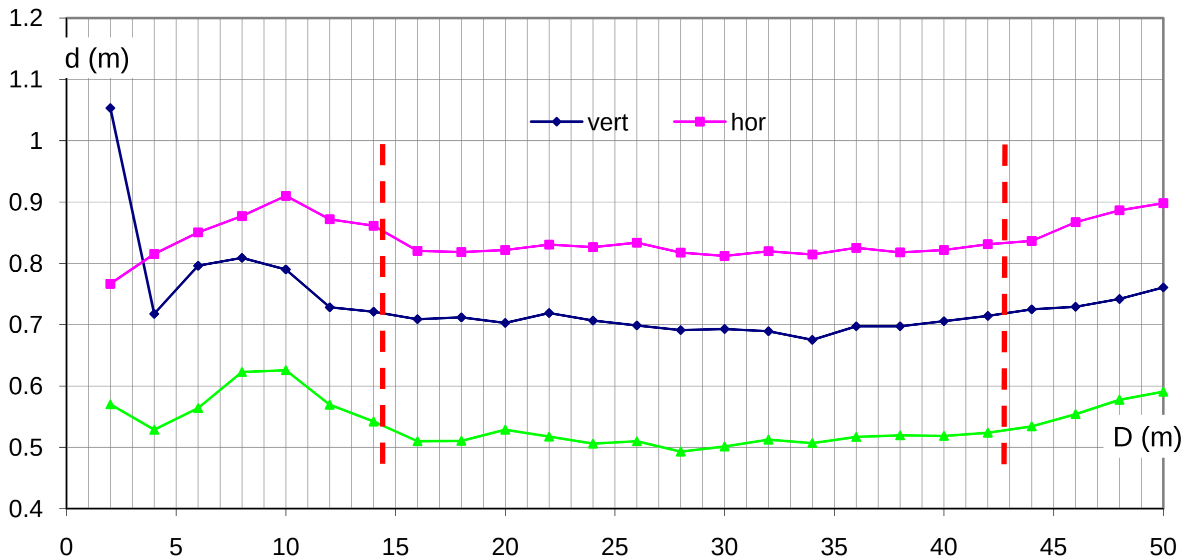


Figure 3: Evolution of the average fractures' spacing on three different directions of survey according to the domain size

Mechanical REV

Mechanical calculations in plane stress conditions were carried out on the fields mentioned above. For this purpose a Finite Element software has been used (ANTHYC developed in the Ecole Polytechnique, France). A Finite element mesh was prepared for each domain from 2 to 20m. Beyond this size, we reach the limits of our numerical tools (maximum number of nodes and finite elements).

Triangular elements are used to make the rock matrix mesh, while the fractures are represented by Goodman quadrangular elements. The evolution of the mesh properties for the various studied samples is illustrated on the figure (4). As an example, the mesh obtained for the three domains of 2, 12 and 20m and the isolines distribution of σ_{xx} stress in a domain of 12m length are presented respectively on figure (5) and (6).

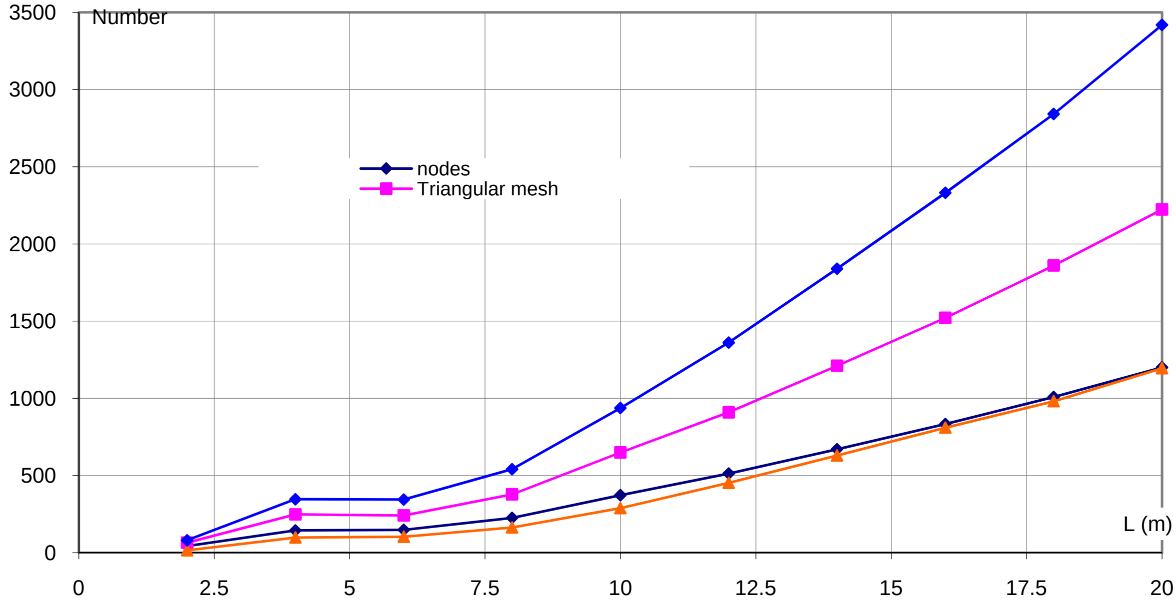


Figure 4: Evolution of the Finite Element mesh properties

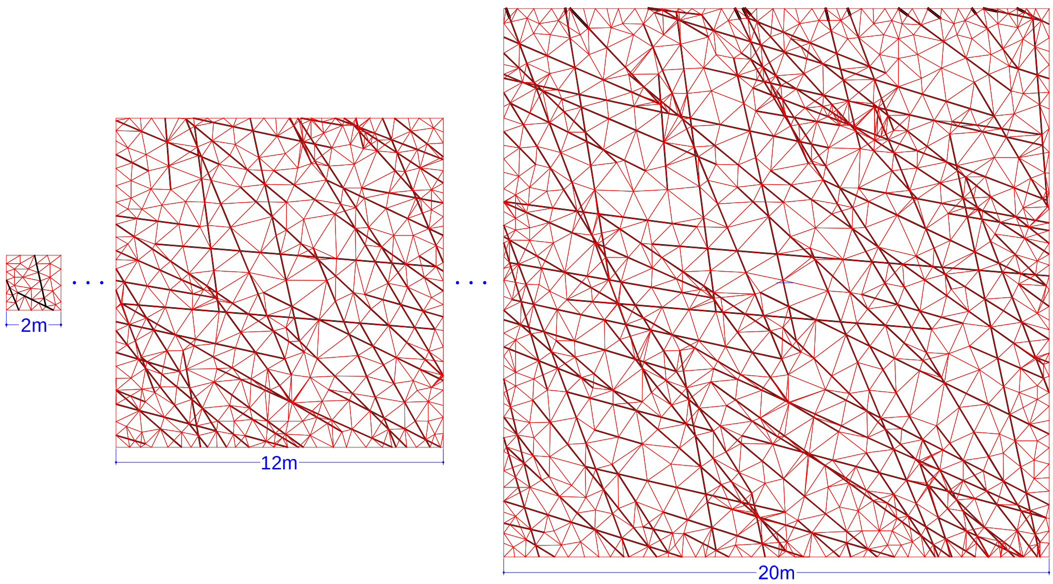


Figure 5: Example of finite element mesh carried out for the fractured fields

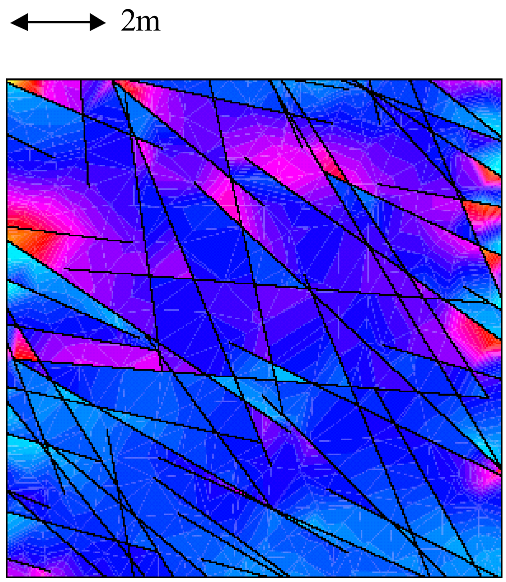
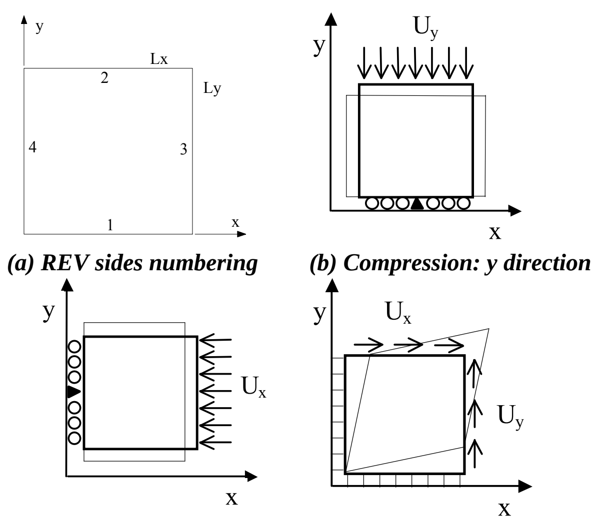


Figure 6: *Example of isolines stresses in x direction according to a simple compression (x direction) on a 12m sample*

Figure 7*a* shows the sides numbering of a rectangular shape; it is used in homogenized stress and strain calculation (Equation 1). On each one of the different domains we carried out three types of loading represented schematically by Figure 7: uniaxial compression in y and x direction (Fig. 7-b and c) and a shear loading in xy direction (Fig. 7*d*). The uniaxial compression in y direction, for instance, is numerically carried out by prescribing a constant displacement Uy=U on the upper side of the domain with Uy=0 on the lower side and surface tension Fx=0 on the lateral sides.



(a) REV sides numbering ***(b) Compression: y direction***

(c) Compression: x direction ***(d) Shear: xy direction***

Figure 7: Numerical experiments (Pouya & Ghoreychi, 2001)

The shear in xy direction is simulated by prescribing a constant displacement Ux=U on the upper side, Ux=0 on the lower side, a constant displacement Uy=U on the right side and Uy=0 on the left side.

The macroscopic homogenized strain and stress, in these cases, can be deduced from the displacements prescribed on the boundary or from the average displacements obtained as a result of calculations on the free boundaries of the domain (Pouya and Ghoreychi 2001). The homogenized stress is calculated as the volume average of stress in all the elements of the domain (equation 1).

Then, we can deduce from the homogenized strains and stresses calculated by equation 1, the Young modulus in different directions, Exx and Eyy, the Poisson's ratio ν_{xy}, ν_{yx} and the Shear modulus G_{xy} (equation 2).

$$\begin{aligned}
\varepsilon_{xx} &= [U_{xx}(4)+U_{xx}(3)]/L_x \\
\varepsilon_{yy} &= [U_{yy}(2)+U_{yy}(1)]/L_y \\
\varepsilon_{xy} &= \{[U_x(2)+U_x(1)]/L_y+[U_y(4)+U_y(3)]/L_x\}/2 \\
\sigma_{xx} &= [F_x(4)+F_x(3)]/2L_y \\
\sigma_{yy} &= [F_y(2)+F_y(1)]/2L_x \\
\sigma_{xy} &= \{[F_x(2)+F_x(1)]/L_x+[F_y(4)+F_y(3)]/L_y\}/4
\end{aligned}$$

(1)

$$\begin{aligned}
&\text{Compression } X: \; E_{xx}=\sigma_{xx}/\varepsilon_{xx}\;,\; \nu_{yx}=\varepsilon_{yy}/\varepsilon_{xx} \\
&\text{Compression } Y: \; E_{yy}=\sigma_{yy}/\varepsilon_{yy}\;,\; \nu_{xy}=\varepsilon_{xx}/\varepsilon_{yy} \\
&\text{Shear } XY \quad : \; G_{xy}=\sigma_{xy}/2\varepsilon_{xy}
\end{aligned}$$

(2)

The results are presented in figure (8). We can notice that all the parameters begin with values near those of intact rock (E=72000MPa, ν=0.25, G=28800Mpa) for small domains. Then the values of Exx, Eyy and Gxy decrease globally with some fluctuations, and attain stable values of ≈65100MPa, 63200MPa and 26800MPa respectively for domain sizes higher than 15m. ν_{xy} and ν_{yx} attain also stable values of ≈0.186 and 0.192 respectively for a size higher than 15m. It is also noted that the evolution of the Young modulus in a direction "i" has a similar shape of the average spacing in the same direction; this effect is not well seen for the Poisson ratio and the Shear modulus. It is important to notice that, for all the domain sizes, the symmetry condition of the elasticity tensor expressed by Exx/ ν_{xy} =Eyy/ ν_{yx} is satisfied by the numerical results with a great accuracy. For instance, for the domain of 18m size, we find Exx=65342MPa, E_{yy}=63207MPa, ν_{xy}=0.192 and ν_{yx}=0.186, and then E_{xx}/ν_{xy}= 340323MPa and E_{yy}/ν_{yx} =339823MPa. This symmetry has not been obtained with the same precision by Min & Jing (ref. 9) probably because of different averaging procedures.

As a conclusion, regarding the results presented in Figure 8 the size of the mechanical REV can be estimated to be about 15m. This size is well comparable to the geometrical REV size.

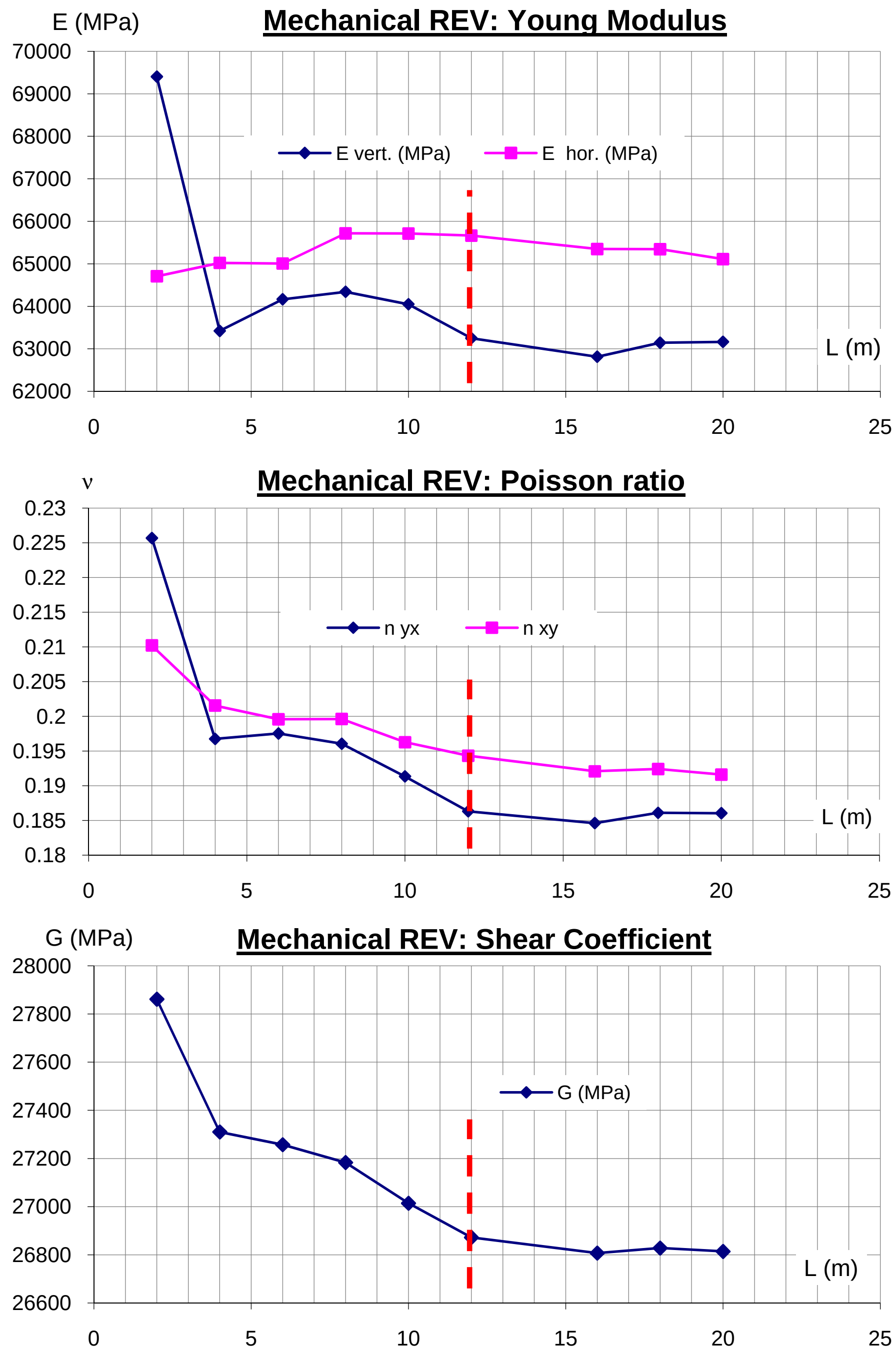


Figure 8: Evolution of the rock mass elastic parameters according to domains size

DISCUSSIONS AND CONCLUSIONS

We thus note that the mechanical and geometrical REV have comparable sizes. This result enables us to estimate in a simple way and less expensive computing time cost, the size of the mechanical REV. Once this size is determined by the study of the geometrical characteristics, then the mechanical characteristics such as the elastic properties or limits of strength can be calculated on a larger or equal size of the REV.

It should however be noted that this result cannot be generalized without precaution.

Initially, this result can be valid only if all the fractures have the same mechanical properties. If the mechanical properties of the fractures are dispersed, the mechanical REV has a different size from the geometrical REV.

Then, the comparison between the geometrical and mechanical REV is only done in one case of FRM, which is not strongly anisotropic. In a strongly anisotropic medium - for instance in the case of one set of fractures - the mechanical or geometrical REV may have different sizes in various directions.

In order to establish general relations between mechanical and geometrical REV size, it is necessary to study many other examples.

ACKNOWLEDGEMENT

The numerical development and technical support by Dr. J. Goambalet from ANDRA (France) is greatly acknowledged.

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